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THE TRANSONIC FLOW OF GAS OVER A CONVEX CORNER *

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The transoic flow of gas over a convex corner with straight line generatrices, in which the Vaglio-Laurin flow /1/ is realized, is considered. This means that in the external potential flow upstream of the corner point all flow parameters are known in the neighborhood of the latter /2—5/. The favorable pressure gradient becomes infinite at the approach to the corner point.

We investigate the interaction between the boundary layer and the external potential flow in the corner point neighborhood, and seek the solution by introducing perturbations in the velocity vector longitudinal component U at the corner point. As the small parameters we take the distance upstream from the corner apex and the reciprocal of the Reynolds number of the oncoming potential stream. Expansions of the flow parameters valid in the basic part of the boundary layer are, then, merged with their expansions in the external potential flow and in the thin boundary layer region next to the wall, which must be taken into account if the boundary conditions are to be satisfied. This makes possible the determination of the behavior of U in the neighborhood of the rectilinear generatrix, which corresponds to the Vaglio-Laurin singularity. As shown in /6—13/, the knowledge of the behavior of U and of the dependence in the external flow on the boundary layer displacement thickness is necessary for determining all characteristic dimensions in the free interaction region.

1. We use the Cartesian system of coordinates x, y whose origin lies at the corner point and the negative semiaxis x coincides with its rectilinear generatrix; v_x and v_y are velocity vector components; φ is the flow potential, p is the pressure, ρ is the density, T is the temperature, a is the speed of sound, and γ is the specific heat ratio; L is a characteristic dimension of the external potential flow, μ is the first coefficient of viscosity, k is the thermal conductivity coefficient; Re and Pr are, respectively, the Reynolds and Prandtl numbers of the oncoming stream. The critical values of all parameters are taken as their characteristic values which are denoted by an asterisk. The thermodynamic variables are related by the equation of state of perfect gas. Below, all flow parameters and equations linking these are assumed to be dimensionless.

The external stream in the region of $x<0$ is potential and defined by Euler's equations. In the corner point neighborhood the solution can be sought in the form /1—5/

$$\varphi = x + y^{7/4} f_0(\xi) + y^{9/4} f_1(\xi) + \ldots, \quad \xi = (1+\gamma)^{-1/3} x y^{-5/4} \tag{1.1}$$

Solution (1.1) for $x<0$, $y=0$ satisfies the impermeability condition $\partial\varphi/\partial y = v_y = 0$ and for $x>0$, $y\to 0$ becomes the Prandtl —Mayer flow. It was also shown in /3—5/ that it is not possible to continue solution (1.1) into the region $x>0$, $y\to 0$ and that it is necessary to introduce a shock wave.

Functions f_0 and f_1 satisfy the ordinary differential equations

$$\left(\frac{25}{16}\xi^2 - f_0'\right) f_0'' - \frac{25}{16}\xi f_0' + \frac{21}{16} f_0 = 0$$

$$\left(\frac{25}{16}\xi^2 - f_0'\right) f_1'' - \left(\frac{45}{16}\xi + f_0''\right) f_1' + \frac{45}{16} f_1 =$$

$$\frac{(1+\gamma)^{-1/3}}{2}\left[(2\gamma-1) f_0'^2 f_0'' + \frac{1}{2}(7f_0 - 5\xi f_0')\left(f_0' - \frac{5}{2}\xi f_0''\right)\right]$$

The Vaglio—Laurin solution f_0 can be represented in the parametric form /2,3/

$$f_0 = C^3 (t-1)^{-7/4} (7t^2 - 140t + 160)/21 \tag{1.2}$$

$$\xi = C(t-1)^{5/4}(t - {}^8/_5), \quad 1 < t < \infty, \quad C = \text{const}$$

The behavior of velocity components v_x and v_y, pressure p, and density ρ at $x<0$ and $y\to 0$ based on solutions (1.2) is defined as follows:

$$v_x = 1 - d_0(-x)^{2/5} - d_1(-x)^{4/5} + \ldots, \quad v_y = -m_0 y (-x)^{-1/5} - m_1 y(-x^{1/5}) + \ldots \tag{1.3}$$

$$\rho = 1 + d_0(-x)^{2/5} + \left[d_1 + \frac{(1-\gamma)}{2} d_0^2\right](-x)^{4/5} + \ldots$$

$$p = 1 + \gamma d_0(-x)^{2/5} + \gamma d_1(-x)^{4/5} + \ldots, \quad m_0 = {}^2/_5(1+\gamma)\, d_0^2$$

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$$d_0 = 3^{3/5}5^{2/5}(1+\gamma)^{-7/15}\ C^{6/5} > 0, \quad m_1 = \text{const}, \quad d_1 = \text{const}$$

Expansions (1.3) imply that the pressure gradient in the neighborhood of the corner point is favorable and

$$\frac{dp}{dx} = -\frac{2}{5}\gamma d_0(-x)^{-3/5} - \frac{4}{5}\gamma d_1(-x)^{-1/5} \to -\infty, \quad (-x)\to 0 \tag{1.4}$$

2. The interaction between the flowing gas and the surface of the corner ($x < 0$) results in the formation of a boundary layer which is subjected to the action of the external flow with the favorable pressure gradient (1.4). We define the boundary layer by equations of conventional form

$$\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho V_y}{\partial Y} = 0 \tag{2.1}$$

$$v_x\frac{\partial v_x}{\partial x} + V_y\frac{\partial v_x}{\partial Y} = -\frac{1}{\gamma\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial Y}\left(\mu\frac{\partial v_x}{\partial Y}\right), \quad \frac{\partial p}{\partial Y} = 0$$

$$\frac{1}{\gamma}v_x\frac{dp}{dx} - \frac{p}{\rho}\left(v_x\frac{\partial\rho}{\partial x} + V_y\frac{\partial\rho}{\partial Y}\right) = \frac{1}{\Pr}\frac{\partial}{\partial Y}\left(k\frac{\partial T}{\partial Y}\right) + \mu(\gamma-1)\left(\frac{\partial v_x}{\partial Y}\right)^2$$

$$Y = \mathrm{Re}^{1/2}y, \quad v_y = \mathrm{Re}^{-1/2}V_y, \quad \mathrm{Re} = \mu_* / (\rho_* a_* L)$$

Below, we assume a linear dependence of the coefficients of viscosity and thermal conductivity on temperature: $\mu = T$ and $k = T$.

The solution of system (2.1) must satisfy the following boundary conditions. When $Y = 0$, $x < 0$ the velocity components $v_x = V_y = 0$ and the corner surface temperature must be either constant or, in the case of a heat insulated wall, $\partial T / \partial Y = 0$. When $Y \to \infty$, $x < 0$ the velocity component v_x and density ρ must merge with expansions (1.3).

Let us assume that there is a solution that satisfies the specified conditions. We denote by $U(Y)$ the profile of the velocity vector longitudinal component at point $x = 0$, $U(Y) = v_x(0, Y)$, and by $R(Y)$ the density, and seek the solution of system (2.1) in the neighborhood of the corner apex in the form of expansions

$$p = 1 + \gamma d_0(-x)^{2/5} + \gamma d_1(-x)^{4/5} + \ldots \tag{2.2}$$

$$v_x = U(Y) + (-x)^{2/5}[u_{00}\ln(-x) + u_{01}] + (-x)^{4/5}[u_{10}\ln^2 \times (-x) + u_{11}\ln(-x) + u_{12}] + \ldots$$

$$V_y = (-x)^{-3/5}[V_{00}\ln(-x) + V_{01}] + (-x)^{-1/5}[V_{10}\ln^2(-x) + V_{11}\ln(-x) + V_{12}] + \ldots$$

$$\rho = R(Y) + (-x)^{2/5}[\rho_{00}\ln(-x) + \rho_{01}] + (-x)^{4/5}[\rho_{10}\ln^2\cdot(-x) + \rho_{11}\ln(-x) + \rho_{12}] + \ldots$$

When $Y \to \infty$ we immediately obtain $R = U = 1$. The functions of Y in expansions (2.2) satisfy systems of ordinary differential equations which can be presented in the general form

$$-\frac{2}{5}(n+1)(\rho_{ni}U + u_{ni}R) + \frac{d}{dY}(RV_{ni}) = P_{ni}^1 \tag{2.3}$$

$$-\frac{2}{5}(n+1)URu_{ni} + U'RV_{ni} = P_{ni}^2$$

$${}^2/_5\,(n+1)\,U\rho_{ni} - R'V_{ni} = P_{ni}{}^3$$

System (2.3) reduces to solving the single equation

$$UV_{ni}' - U'V_{ni} = [U(P_{ni}^1 + P_{ni}^3) - P_{ni}^2]/R \tag{2.4}$$

Having solved Eqs. (2.4) we determine functions u_{ni} and ρ_{ni} using formulas

$$u_{ni} = \frac{5}{2(n+1)}\left[\frac{U'}{U}V_{ni} - \frac{P_{ni}^2}{UR}\right], \quad \rho_{ni} = \frac{5}{2(n+1)}\left[\frac{R'}{U}V_{ni} + \frac{P_{ni}^3}{U}\right] \tag{2.5}$$

For $i = 0$, $n = 0$ system (2.3) is homogeneous and its solution is of the form

$$V_{00} = A_{00}U(Y), \quad \rho_{00} = {}^5/_2\,A_{00}R'(Y), \quad u_{00} = {}^5/_2 A_{00}U'(Y) \tag{2.6}$$

For $n = 0$, $i = 1$ the solution of system (2.4), (2.5) can be represented in the form

$$V_{01} = \left[A_{01} + \frac{2}{5}d_0 I(Y)\right]U(Y) \tag{2.7}$$

$$U_{01} = \left[\frac{5}{2}A_{01} - \frac{25}{4}A_{00} + d_0 I(Y)\right]U'(Y) - \frac{d_0}{UR}$$

$$\rho_{01} = \left[\frac{5}{2}A_{01} - \frac{25}{4}A_{00} + d_0 I(Y)\right]R'(Y) + d_0 R$$

$$I(Y) = \int_Y^\infty \frac{1 - U^2R}{U^2R}\, dY$$

The behavior of $U(Y)$ and $R(Y)$ as $Y \to \infty$ is assumed such that the integral $I(Y)$ is convergent. The solution for functions with subscripts $n = 1$ and $i = 0$ is of the form

$$\begin{aligned} V_{10} &= A_{10}U(Y) + {}^5/_2\, A_{00}{}^2 U'(Y) \\ u_{10} &= {}^5/_4\, [A_{10}U'(Y) + {}^5/_2\, A_{00}{}^2\, U''(Y)] \\ \rho_{01} &= {}^5/_4\, [A_{10}\, R'(Y) + {}^5/_2\, A_{00}{}^2\, R''(Y)] \end{aligned} \tag{2.8}$$

The solutions for functions with subscripts $n = 1$ and $i = 1, 2$ are not presented here owing to their unwieldiness. Note that for $n = 1$ and $i = 1$ the right-hand sides of system (2.3) approach zero as $Y \to \infty$, which means that

$$\lim V_{11} = A_{11},\ \lim u_{11} = \lim \rho_{11} = 0,\ Y \to \infty \tag{2.9}$$

In the case of $n = 1$ and $i = 2$ we have

$$\begin{aligned} &\lim P_{12}{}^2 = {}^4/_5\, d_1,\ \lim P_{12}{}^3 = {}^4/_5\, d_1 + {}^2/_5\, (1 - \gamma)\, d_0{}^2 \\ &\lim \{[(P_{12}{}^2 + P_{12}{}^3)\, U - P_{22}{}^2] / R\} = -{}^2/_5\, (1 + \gamma)\, d_0{}^2,\ Y \to \infty \end{aligned}$$

from which and Eqs. (2.4) and (2.5) follows

$$\begin{aligned} V_{12} &= A_{12} - {}^2/_5\, (1 + \gamma)\, d_0{}^2 Y + o(1),\ u_{12} = -d_1 + o(1) \\ \rho_{12} &= d_1 + {}^1/_2\, (1 - \gamma)\, d_0{}^2 + o(1),\ Y \to \infty \end{aligned} \tag{2.10}$$

Note that in the considered approximation the right-hand sides of system (2.3) do not contain dissipative terms. Hence the flow in the main part of the boundary layer is vortical, and it is possible to neglect in it the effects of dissipative factors and represent it by expansions (2.2).

Finally, we merge expansions (1.3) and (2.2). The external variable y is related to the internal Y by formula $y = \mathrm{Re}^{-1/2}\, Y$. Formulas (2.6) — (2.10) imply that the external expansion of the internal expansion (2.2) for v_x and ρ fully match their expansions (1.3) in the external potential stream. For v_y we have

$$\begin{aligned} v_y = &-{}^2/_5\, (1 + \gamma)\, d_0{}^2 y + \mathrm{Re}^{-1/2}\, \{[A_{00} \ln(-x) + A_{01}]\, (-x)^{-3/5} + \\ &O\, [(-x)^{-1/5} \ln(-x)]\} \end{aligned} \tag{2.11}$$

We thus find that for merging in the first approximation v_y in the potential stream it is necessary to consider in expansions (2.2) terms of order up to $(-x)^{-1/5}$.

3. However it is not possible to satisfy the boundary conditions at the corner surface, using expansion (2.2). Because of this it is necessary to introduce in the boundary layer region next to the wall a thin sublayer in which viscosity plays a predominant part. As implied by (2.2) the boundary layer displacement thickness is defined by $\delta \sim \mathrm{Re}^{-1/2}\, (-x)^{2/5} \ln(-x)$. The effect of heat conduction on the flow pattern is minor, since under the specified thermal conditions at the corner surface and low velocities of motion the compressibility of gas manifests itself only weakly. For the sublayer we seek a solution of the form

$$\begin{aligned} &v_x = (-x)^{1/5} u_0(\eta) + (-x)^{3/5} u_1(\eta) + \ldots,\ \rho = R(0) + \\ &\quad (-x)^{2/5} \rho_1(\eta) + \ldots \\ &V_y = (-x)^{-2/5} V_0(\eta) + V_1(\eta) + \ldots \\ &p = 1 + \gamma d_0\, (-x)^{2/5} + \gamma d_1\, (-x)^{4/5} + \ldots,\quad \eta = Y / (-x)^{2/5} \end{aligned} \tag{3.1}$$

The exponents of $(-x)$ in the first terms of expansions of v_x and V_y, and of the self-similar variable η are determined by the condition that along lines $\eta = \mathrm{const}$ the terms of continuity and of motion in Eqs. (2.1) must be of the same order. This is equivalent to the requirement that the forces of friction, inertia, and pressure must play equal parts in shaping the flow in the sublayer. The temperature is determined by the equation of state

$$T = \frac{1}{R(0)} \left[1 + (-x)^{2/5} \left(\gamma d_0 - \frac{\rho_1}{R(0)} \right) + \ldots \right]$$

We introduce the stream function

$$\Psi = (-x)^{3/5}\, F_0(\eta) + (-x)\, F_1(\eta) + \ldots$$

Functions F_0 and F_1, and the velocity components are related by formulas

$$\begin{aligned} &u_0 = F_0{}',\quad V_0 = {}^3/_5\, F_0 - {}^2/_5\, \eta F_0{}',\quad u_1 = F_1{}' - \frac{\rho_1 u_0}{R(0)} \\ &V_1 = F_1 - {}^2/_5\, \eta F_1{}' - \rho_1 V_0 / R(0) \end{aligned} \tag{3.2}$$

For determining the first approximations functions we have for F_0 the equations

$$-\frac{1}{R^2(0)}\frac{d^3F_0}{d\eta^3}+\frac{3}{5}F_0\frac{d^2F_0}{d\eta^2}-\frac{1}{5}\left(\frac{dF_0}{d\eta}\right)^2=\frac{2}{5}\frac{d_0}{R(0)} \tag{3.3}$$

and for the second approximation of functions F_1 and ρ_1 we have the system

$$-\frac{1}{R^2(0)}F_1'''+\frac{3}{5}F_0F_1''-\frac{4}{5}F_0'F_1'+F_0''F_1-\frac{4}{5}\frac{d_1}{R(0)}=$$
$$\frac{1}{R(0)}\left\{\frac{3}{5}\left[-\rho_1F_0'^2+F_0\frac{d}{d\eta}(\rho_1F_0')\right]+\right.$$
$$\left.\frac{1}{R(0)}\frac{d}{d\eta}\left[\left(\gamma d_0-\frac{\rho_1}{R(0)}\right)F_0''\right]\right\}=E_1 \tag{3.4}$$
$$\frac{1}{R^2(0)\,\mathrm{Pr}}\rho_1''+\frac{2}{5}u_0(\rho_1-\eta\rho_1')-V_0\rho_1'=(\gamma-1)u_0'^2+$$
$$\frac{2}{5}R(0)\,d_0u_0=N_1$$

For Eqs. (3.3) and (3.4) we have the following boundary conditions. As $\eta\to\infty$ expansions (3.1) must merge with expansions (2.2). For $Y=0$ we have $F_0=F_0'=F_1=F_1'=0$. When the temperature of the corner surface is constant, then $\rho_1(0)=\gamma R(0)$, if however the corner surface is thermally insulated, then $\rho'_1(0)=0$. Note that Eqs. (3.4) are linear and the solution of the second of these is independent of the first.

When $\eta\to 0$ the solutions for F_0, ρ_1 and F_1 can be represented in the form

$$F_0=\sum_{n=0}^{\infty}\beta_n\eta^{n+2},\quad \rho_1=\sum_{n=0}^{\infty}\omega_n\eta^n,\quad F_1=\sum_{n=0}^{\infty}\varkappa_n\eta^{n+2} \tag{3.5}$$

where the coefficient β_0 is arbitrary, $\beta_1=-15^{-1}d_0R^2(0)$, $\beta_2=0$, and the remaining $\beta_n\ (n\geqslant 2)$ are determined by β_0 and β_1. The coefficients ω_0 and ω_1 are arbitrary, and the remaining $\omega_n\ (n\geqslant 2)$ are determined by ω_0, ω_1, β_0 and β_1. The quantity $\varkappa_0$ is also arbitrary, and $\varkappa_n\ (n\geqslant 1)$ are expressed in terms of $\varkappa_0$, β_0, β_1, ω_0 and ω_1.

The asymptotic behavior of solution F_0, as $\eta\to\infty$, is of the form

$$F_0=B_0\eta^{3/2}+B_{00}\eta^{1/2}\ln\eta+B_{01}\eta^{1/2}+\ldots$$
$$B_{00}=-\frac{2}{3}\frac{d_0}{B_0R(0)}, \tag{3.6}$$

Functions ρ_1 and F_1 are obtained by solving inhomogeneous equations. As $\eta\to\infty$, the right-hand sides of E_1 and N_1 behave as $O(\eta)$ and $O(\sqrt{\eta})$. The asymptotic behavior of ρ_1 and F_1, as $\eta\to\infty$ conforms to

$$\rho_1=C_1\eta-\frac{4}{9}\frac{d_0C_1}{B_0^2R(0)}\ln\eta+C_{01}\ldots \tag{3.7}$$
$$F_1=M_1\eta^{5/2}+M_{10}\eta^{3/2}\ln\eta+M_{11}\eta^{3/2}+\ldots$$

The properties of the asymptotic expansions of F_0 and F_1 for $\eta\to 0$ and $\eta\to\infty$ obtained here coincide with those in /14,15/. In the expansions of solutions (3.6) and (3.7) for F_0, F_1, and ρ_1 only the number of terms necessary for merging expansions (3.1) with the terms of expansions (2.2) of order $(-x)^{2/5}$. If $(-x)^{2/5}$ is assumed small, the external variable Y is related to the internal η by the formula $\eta=Y/(-x)^{2/5}$.

The external expansion of the internal expansion represented in terms of external variables is of the form

$$v_x={}^3/_2B_0Y^{1/2}+\frac{5}{2}MY^{3/2}-{}^1/_5B_{00}Y^{-1/2}(-x)^{2/5}\ln(-x)+$$
$$[{}^1/_2B_{00}\ln Y+(B_{00}+{}^1/_2B_{01})]\,Y^{-1/2}(-x)^{2/5}+\ldots,\quad \rho=R(0)+$$
$$C_1Y+\ldots$$
$$M=M_1-\frac{3}{5}C_1B_0,$$
$$V_y=-{}^4/_{25}B_{00}Y^{1/2}(-x)^{-3/5}\ln(-x)+$$
$$[{}^2/_5B_{00}\ln Y+{}^2/_5(B_{01}-B_{00})]Y^{1/2}(-x)^{-3/5}+\ldots$$

Comparison with the internal expansion of the external expansion (2.2), expressed in terms of external variables yields the relation between constants and the behavior of functions $U(Y)$ and $R(Y)$ as $Y\to 0$. We have

$$A_{00}=\left(\frac{4}{15}\right)^2\frac{d_0}{B_0^2R(0)},\quad A_{01}=\frac{4}{15}\frac{d_0}{B_0^2R(0)}-\frac{2}{5}b_0d_0$$
$$b_0=\int_0^{\infty}\frac{1-U^2R}{R}\left[U^{-2}-\frac{4}{9B_0^2}Y^{-1}\right]dY-$$

$$\frac{4}{9B_0^2}\int_0^\infty \ln Y \frac{d}{dY}\left[\frac{1-U^2R}{R}\right] dY$$

$$U(Y) = \frac{3}{2} B_0 Y^{1/2} + \frac{5}{2} M Y^{3/2} + \ldots, \quad R(Y) = R(0) + C_1 Y + \ldots$$

4. Let us revert to expansion (2.11). It is evident that owing to the displacing effect of the boundary layer (more exactly, of its sublayer) it is necessary to introduce in expansion (1.1) for the external stream terms proportional to $\mathrm{Re}^{-1/2}$

$$\varphi = x + y^{7/4} f_0(\xi) + y^{9/4} f_1(\xi) + \mathrm{Re}^{-1/2}\left[y^{1/4} \ln y\, f_{-10}(\xi) + y^{1/4} f_{-11}(\xi)\right] + \ldots = \varphi_0 + \mathrm{Re}^{-1/2}\varphi_{-1}$$

The function f_{-11} satisfies the equation

$$\left(\frac{25}{16}\xi^2 - f_0'\right) f_{-11}'' - \left[\frac{35}{16}\xi + f_0''\right] f_{-11}' - \frac{3}{16} f_{-11} = -\frac{1}{4} f_{-10} + \frac{5}{4}\xi f_{-10}' \tag{4.1}$$

Function f_{-10} satisfies the homogenous equation (4.1). As $\xi \to -\infty$ the asymptotic behavior of f_{-10} and f_{-11} is defined by the expansions

$$f_{-10} = D_{-10}^{s}\left[\xi^{1/5} + O(\xi^{-7/5})\right] + D_{-10}^{a}\left[\xi^{-3/5} + O(\xi^{-11/5})\right]$$

$$f_{-11} = D_{-11}^{s}\left[\xi^{1/5} + O(\xi^{-7/5})\right] + D_{-11}^{a}\left[\xi^{-3/5} + O(\xi^{-11/5})\right] + {}^{4}/_{5}\, D_{-10}^{a}\xi^{-3/5}\ln\xi$$

The superscripts s and a correspond, respectively, to the symmetric and antisymmetric solutions. Using the obtained expansions we obtain the asymptotic behavior of velocity components for $x < 0,\ y \to 0$, which correspond to the potential φ_{-1}

$$\begin{aligned}\mathrm{Re}^{-1/2}\frac{\partial\varphi_{-1}}{\partial y} &= \left(-\frac{x}{\beta}\right)^{1/5} D_{-10}^{s} Y^{-1} + \frac{4}{5}\mathrm{Re}^{-1/2} D_{-10}^{a}\left(-\frac{x}{\beta}\right)^{-3/5}\ln\left(-\frac{x}{\beta}\right) + \\ &\mathrm{Re}^{-1/2} D_{-11}^{a}\left(-\frac{x}{\beta}\right)^{-3/5} + O(\mathrm{Re}^{-1}) \\ \mathrm{Re}^{-1/2}\frac{\partial\varphi_{-1}}{\partial x} &= -\mathrm{Re}^{-1/2}\ln \mathrm{Re}^{-1/2}\frac{1}{5\beta} D_{-10}^{s}\left(-\frac{x}{\beta}\right)^{-4/5} - \\ &\mathrm{Re}^{-1/2}\left[\frac{1}{5\beta} D_{-10}^{s}\ln Y + \frac{1}{5\beta} D_{-11}^{s}\right]\left(-\frac{x}{\beta}\right)^{-4/5} + O(\mathrm{Re}^{-1}) \\ \beta &= (1+\gamma)^{1/3}\end{aligned} \tag{4.2}$$

It is obvious that for the expansion of pressure in the main part of the boundary layer to be independent of coordinate Y it is necessary to set $D_{-10}^{s} = 0$.

We seek a solution for the main part of the boundary layer, induced by the potential $\mathrm{Re}^{-1/2}\varphi_{-1}$, of the form

$$\begin{aligned} v_x &= U(Y) + O\left[(-x)^{2/5}\ln(-x)\right] + \mathrm{Re}^{-1/2}\left[u_{-10}\ln(-x) + u_{-11}\right](-x)^{-4/5} + \ldots \\ V_y &= O\left[(-x)^{-3/5}\ln(-x)\right] + \mathrm{Re}^{-1/2}\left[V_{-10}\ln(-x) + V_{-11}\right]\times (-x)^{-3/5} + \ldots \\ \rho &= R(Y) + O\left[(-x)^{2/5}\ln(-x)\right] + \mathrm{Re}^{-1/2}\left[\rho_{-10}\ln(-x) + \rho_{-11}\right](-x)^{-4/5} + \ldots \\ p &= 1 + O\left[(-x)^{2/5}\right] + \gamma\,\mathrm{Re}^{-1/2} d_{-1}(-x)^{-4/5} + \ldots \\ d_{-1} &= {}^{1}/_{5}(1+\gamma)^{-1/15} D_{-11}^{s}\end{aligned} \tag{4.3}$$

The unknown functions in (4.3) satisfy the system of Eqs. (2.3) whose solutions are

$$V_{-10} = A_{-10}U(Y), \quad \rho_{-10} = -{}^{5}/_{4}A_{-10}R'(Y), \quad u_{-10} = -{}^{5}/_{4}A_{-10}U'(Y)$$

$$u_{-11} = -\left[\frac{5}{4}A_{-11} + \frac{25}{16}A_{-10} - d_{-1}I(Y)\right]U(Y) - \frac{d_{-1}}{UR}$$

$$\rho_{-11} = -\left[\frac{5}{4}A_{-11} + \frac{25}{16}A_{-10} - d_{-1}I(Y)\right]R'(Y) + d_{-1}R$$

$$V_{-11} = \left[A_{-11} - \frac{4}{5}d_{-1}I(Y)\right]U(Y)$$

Using these solutions it is possible to show that expansions (4.3) merge with the expansions of v_x and ρ in the external potential stream and induce perturbations of the potential

$$\mathrm{Re}^{-1}y^{-6/4}\left[f_{-20}(\xi)\ln y + f_{-21}(\xi)\right]$$

To satisfy the boundary conditions at the corner surface it is necessary, as previously, to introduce the viscous sublayer. We seek for it a solution of the form

$$\begin{aligned} v_x &= (-x)^{1/5}u_0 + (-x)^{3/5}u_1 + \mathrm{Re}^{-1/2}(-x)^{-1}u_{-1} + \ldots \\ V_y &= (-x)^{-2/5}V_0 + V_1(\eta) + \mathrm{Re}^{-1/2}(-x)^{-8/5}V_{-1} + \ldots \\ \rho &= R(0) + (-x)^{2/5}\rho_1(\eta) + \mathrm{Re}^{-1/2}(-x)^{-4/5}\rho_{-1}(\eta) + \ldots\end{aligned} \tag{4.4}$$

We introduce function F_{-1} defined by formulas

$$u_{-1} = F_{-1}', \ V_{-1} = -{}^3/_5\, F_{-1} - {}^2/_5\, \eta\, F_{-1}'$$

and obtain for the determination of solution in the sublayer the following system:

$$-\frac{1}{R^2(0)} F_{-1}''' + \frac{3}{5} F_0 F_{-1}'' + \frac{4}{5} F_0' F_{-1}' - \frac{3}{5} F_0'' F_{-1} = -\frac{4}{5}\frac{\alpha_1}{R(0)} \tag{4.5}$$

$$\frac{1}{R^2(0)\,\mathrm{Pr}} \rho_{-1}'' - V_0 \rho_{-1} - u_0\left(-\frac{6}{5}\rho_{-1} + \frac{2}{5}\eta\rho_{-1}'\right) =$$
$$R(0)\left[-\frac{4}{5} d_{-1} u_0 + \frac{2}{5} d_0 u_{-1}\right] + u_{-1}\left[-\frac{2}{5}\rho_1 + \frac{2}{5}\eta\rho_1'\right] +$$
$$V_{-1}\rho_1' + 2(\gamma - 1) u_0' u_{-1}'$$

The asymptotic behavior of solutions for F_{-1} and ρ_{-1} as $\eta \to 0$ is the same as that of F_1 and ρ_1. As $\eta \to \infty$ we have

$$F_{-1} = M_{-1}\eta^{-3/2} - \frac{2}{3}\frac{d_{-1}}{B_0 R(0)}\eta^{1/2}\ln\eta + \ldots, \quad \rho_{-1} = C_{-1}\eta^{-2} + \ldots$$

The merging of expansions (4.4) with expansions that are valid in the basic part of the boundary layer and in the external potential flow yields relationships for contants in solutions and new terms in expansions of $v(Y)$ and $R(Y)$ as $Y \to 0$

$$U(Y) = {}^3/_2\, B_0 Y^{1/2} + {}^5/_2\, M Y^{3/2} - {}^3/_2\, \mathrm{Re}^{-1/2} Y^{-5/2}\, M_{-1} + \ldots$$
$$R(Y) = R(0) + C_1 Y + \mathrm{Re}^{-1/2}\, C_{-1} Y^{-2} + \ldots$$
$$A_{00} = {}^4/_5\,(1+\gamma)^{1/5}\, D_{-10}^a,\ A_{10} = -{}^4/_5\,(1+\gamma)^{1/5} \ln\beta\, D_{-10}^a +$$
$$(1+\gamma)^{1/5}\, D_{-11}^a$$
$$A_{-10} = -\frac{32}{225}\frac{d_{-1}}{B_0^2 R(0)}, \quad A_{-11} = \frac{4}{5} d_{-1} b_0 + \frac{8}{45}\frac{d_{-1}}{B_0^2 R(0)}$$

5. The comparison of terms in expansions (4.3) for pressure and expansions (4.4) along lines $\eta = \mathrm{const}$ show that the terms related to the displacing effect of the boundary layer are at distances $(-x) \sim \mathrm{Re}^{-5/12}$ of the same order as the terms define the effect of the external potential stream. This means that in the neighborhood of the corner apex there is a region of free interaction that corresponds to the Vaglio-Laurin singularity. This result can be also obtained in another way /12,13/. For this it is necessary to know the behavior of $U(Y)$ as $Y \to 0$ and, also, the link between the boundary layer displacement thickness and the pressure induced by it.

Let the basic profile $U(Y) = O(Y^z)$ for $Y \to 0$ and $z > 0$. The free interaction region has a three-layer structure /6—9/. A viscous incompressible sublayer lies next to the wall; in it the forces of pressure, inertia, and friction mutually balance themselves. We denote by a vinculum the variables and parameters of the stream whose order of magnitude in the sublayer is comparable with unity. Taking into account the equations of continuity and momenta (2.1) we obtain

$$x = \mathrm{Re}^{-\alpha}\bar{x}, \quad Y = \mathrm{Re}^{-\beta}\bar{Y}, \quad \Delta p = \mathrm{Re}^{-\tau}\bar{p}, \quad v_x = \mathrm{Re}^{-\beta z}\bar{v}_x$$
$$V_y = \mathrm{Re}^{\alpha - \beta(1+z)}\bar{V}_y, \quad \tau = 2\beta z, \quad \alpha = \beta(2 + z) \tag{5.1}$$

The solution for the main part of the boundary layer is sought in the form

$$v_x = U(Y) + \mathrm{Re}^{-\varkappa} u_0(x, Y) + \ldots, \quad V_y = \mathrm{Re}^{-\varkappa + \alpha} v_0(x, Y) \tag{5.2}$$

We have to determine in formulas (5.1) and (5.2) the exponents α, β, τ, and $\varkappa$. In the transonic velocity range the flow deflection angle θ is related to the relative pressure variation Δp by formula $\Delta p = O(\theta^{2/3})$ and is determined by the displacing action of the viscous sublayer. From this we obtain the missing formulas for $\alpha, \beta, \tau, \varkappa$: $\varkappa = \beta, \varkappa - \alpha + {}^1/_2 = 3\tau/2$. We have

$$\alpha = \frac{2+z}{2(1+4z)}, \quad \beta = \frac{1}{2(1+4z)}, \quad \varkappa = \beta, \quad \tau = \beta$$

which for $z = {}^1/_2$ yield $\alpha = {}^5/_{12}, \beta = {}^1/_6$. For $z = 1$ we have the Blasius profile $\alpha = {}^3/_{10}, \beta = {}^1/_{10}$, which conforms to the results in /13/.

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